Exotic few-body systems with a heavy meson

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Outline

1 Introduction
   - Hadronic molecule
   - Heavy Quark Spin Symmetry and one pion exchange potential

2 Results of $\bar{D}N$ and $BN$

3 Results of $\bar{D}NN$ and $BNN$

4 Results in the heavy quark limit ($m_Q \to \infty$)

5 Summary
**Exotic hadrons in the heavy quark region**

**Introduction**

- Constituent quark model has been successfully applied to hadron spectra. : $qqq$ and $q\bar{q}$  **(Ordinary hadrons)**
  

- **New Exotic hadrons** $X$, $Y$, $Z$: KEK, SLAC, ...
  
  (in the heavy quark $(c, b)$ sector)

  $\Rightarrow$ They cannot be explained by the simple quark model.
Constituent quark model has been successfully applied to hadron spectra: \(qqq\) and \(q\bar{q}\) (Ordinary hadrons)


New Exotic hadrons \(X, Y, Z\): KEK, SLAC, ...
(in the heavy quark \((c, b)\) sector)
⇒ They cannot be explained by the simple quark model.

How can we understand structures of the exotic hadrons?
Exotic hadrons in the heavy quark region

Introduction

- Constituent quark model has been successfully applied to hadron spectra. : $qqq$ and $q\bar{q}$  (Ordinary hadrons)
  

**Baryon**

**Meson**

**Multiquark states? (Meson)**

- Tetraquark (Compact)
  - $Q$: Heavy quark ($c, b$)
  - $q$: Light quark ($u, d$)

- Hadronic molecule
  - $Q$: Heavy quark ($c, b$)
  - $\bar{Q}$: Heavy antiquark ($c, b$)
  - $q$: Light quark ($u, d$)
  - $\bar{q}$: Light antiquark ($u, d$)

- $D^*$
- $\bar{D}$

Candidates: $X(3872)\equiv(D\bar{D})$, $Z_b\equiv(B\bar{B})$...


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Exotic hadrons in the heavy quark region

**Introduction**

- Constituent quark model has been successfully applied to hadron spectra. : $qqq$ and $q\bar{q}$ (Ordinary hadrons)


- **Baryon**
  - **Meson**
  - **Multiquark states? (Meson)**

- **Tetraquark (Compact)**
  - $Q$: Heavy quark ($c, b$), $q$: Light quark ($u, d$)

- **Candidates?** $X(3872)(D\bar{D}^*?)$, $Z_b(B\bar{B}^*?)$...

Hadronic molecule
Introduction

Hadronic molecules

- Loosely bound states (resonances) of hadrons
  → Analogous to Deuteron (proton-neutron)

Meson-Meson
Meson-Baryon


In the molecules, information on Hadron-Hadron interactions are important!
Hadronic molecule
Introduction

Loosely bound states (resonances) of hadrons
→ Analogous to **Deuteron (proton-neutron)**

Meson-Meson molecules: $X(3872)$ ($D\bar{D}^*$), $Z_b$ ($B\bar{B}^*$)...

Hadronic molecule

Introduction

Loosely bound states (resonances) of hadrons
→ Analogous to Deuteron (proton-neutron)

Meson-Meson molecules: X(3872) (D\bar{D}^*), Z_b (B\bar{B}^*)...

Meson-Nucleon molecules: Λ(1405) (\bar{K}N), Λ_\bar{c}^* (D\bar{N})...

⇒ In the molecules, information on Hadron-Hadron interactions are important!
Interaction in the heavy flavor sector

Introduction

Strange (Light)

\[ K \rightarrow N, \rho, \omega, \ldots \]

Charm (Heavy)

\[ \bar{D} \rightarrow N, \rho, \omega, \ldots \]

Short range force (\( \rho, \omega \) exchanges...) dominates in \( KN \).
Interaction in the heavy flavor sector

Introduction

- Strange (Light)
  - $K N$
    - $\rho, \omega, \ldots$.

- Charm (Heavy)
  - $\bar{D} N$
    - $\pi, \rho, \omega, \ldots$
  - $\bar{D}^* N$
    - $\pi, \rho, \omega, \ldots$

- Short range force ($\rho, \omega$ exchanges...) dominates in $KN$.
- In the heavy sector, $\bar{D} - \bar{D}^*$ mixing caused by small $\Delta m_{\bar{D} \bar{D}^*}$ enhances one $\pi$ exchange potential (OPEP).
- The small mass splitting is induced by the Heavy Quark Spin Symmetry!
Heavy Quark Spin Symmetry (HQSS)  

- HQSS appears in the heavy quark mass limit \( (m_Q \to \infty) \).
- Spin-spin interaction \( \to 0 \)

\[
\begin{align*}
\text{e.g. Heavy meson } (\bar{Q}q) \\
\begin{array}{c}
P^* \\
P
\end{array}
\end{align*}
\]

Heavy pseudoscalar meson \( P(0^-) \) and Heavy vector meson \( P^*(1^-) \) are degenerate.

Indeed, mass splitting between \( P \) and \( P^* \) is small.

\[
\begin{align*}
&\begin{align*}
&m_{B^*} - m_B \sim 45 \text{ MeV} \\
&m_{D^*} - m_D \sim 140 \text{ MeV}
\end{align*} \\
\iff
&\begin{align*}
&m_{K^*} - m_K \sim 400 \text{ MeV}
\end{align*}
\end{align*}
\]

For strangeness sector.
Heavy Quark Spin Symmetry (HQSS) N. Isgur, M. B. Wise, PRL 66, 1130

- HQSS appears in the heavy quark mass limit \( (m_Q \to \infty) \).
- Spin-spin interaction \( \to 0 \)

  e.g. Heavy meson \( (\bar{Q}q) \)

\[
\begin{align*}
& \left\{ \begin{array}{c}
\text{Heavy pseudoscalar meson } P(0^-) \text{ and } \\
\text{Heavy vector meson } P^*(1^-) \text{ are degenerate.}
\end{array} \right.
\end{align*}
\]

Indeed, mass splitting between \( P \) and \( P^* \) is small.

\[
\left\{ \begin{array}{c}
m_{B^*} - m_B \sim 45 \text{ MeV} \\
m_{D^*} - m_D \sim 140 \text{ MeV}
\end{array} \right. \quad \Leftrightarrow \quad \text{For strangeness sector } \ m_{K^*} - m_K \sim 400 \text{ MeV}
\]

▷ OPEP appears through \( PP^*\pi \) and \( P^*P^*\pi \) vertices. \((PP\pi \text{ is forbidden.})\)
▷ Thanks to the degeneracy, OPEP is enhanced.
Lagrangian($P(\ast) - N$) and Form factor

- **Lagrangian**
  - **Heavy-light chiral Lagrangian**
    
    $\mathcal{L}_{\pi HH} = ig_\pi \text{Tr} \left[ H_b \gamma_\mu \gamma_5 A_{ba}^\mu \bar{H}_a \right], \quad g_\pi = 0.59$ for $D$ and $B$
    
    From $D^* \to D\pi$ decay

    $H_a = \frac{1 + \gamma^\nu}{2} \left[ P_a^* \gamma_\mu - P_a \gamma^5 \right], \quad \bar{H}_a = \gamma^0 H_a \gamma^0$

    vector, pseudoscalar

  - **Bonn model**
    
    $\mathcal{L}_{\pi NN} = ig_\pi NN \bar{N}_b \gamma^5 N_a \hat{\pi}_{ba}, \quad g^2_{\pi NN}/4\pi = 13.6$
    
    From NN data


- R. Machleidt *et al.* Phys Rept. 149 (1987) 1
Lagrangian($P^(*) - N$) and Form factor

\subsection*{Lagrangian}

\begin{itemize}
  \item Heavy-light chiral Lagrangian \hspace{1cm} R.Casalbuoni et al. PhysRept.\textbf{281}(1997)145

  \begin{align*}
  \mathcal{L}_{\pi HH} &= ig_{\pi} \text{Tr} \left[ H_b \gamma_{\mu} \gamma_5 A_{ba}^\mu \bar{H}_a \right], \quad g_{\pi} = 0.59 \text{ for } \bar{D} \text{ and } B \\
  H_a &= \frac{1 + \not{\gamma}}{2} \left[ P_a^* \gamma_{\mu} - P_a \gamma^5 \right], \quad \bar{H}_a = \gamma^0 H_a \gamma^0
  \end{align*}

  \text{From } D^* \rightarrow D\pi \text{ decay}

  \item Bonn model \hspace{1cm} R.Machleidt et al. Phys Rept.\textbf{149}(1987)1

  \begin{align*}
  \mathcal{L}_{\pi NN} &= ig_{\pi NN} \bar{N}_b \gamma^5 N_a \pi_{ba}, \quad g_{\pi NN}^2 / 4\pi = 13.6
  \end{align*}

  \text{From } NN \text{ data}

\end{itemize}

\subsection*{Form factor ($P^{(*)} N$)}

\begin{align*}
  F(\vec{q}) &= \frac{\Lambda_N^2 - m_N^2}{\Lambda_N^2 + |\vec{q}|^2} \frac{\Lambda_P^2 - m_P^2}{\Lambda_P^2 + |\vec{q}|^2}
\end{align*}

\begin{enumerate}
  \item \(\Lambda_N\) is fixed to reproduce the Deuteron. \((NN \text{ system})\)
  \item \(\Lambda_P\); We assume \(\Lambda_P / \Lambda_N = r_N / r_P\).
\end{enumerate}

\begin{align*}
  \begin{cases}
  \Lambda_D = 1.35 \Lambda_N & \Rightarrow \quad \Lambda_N = 830 \text{ MeV} \\
  \Lambda_B = 1.29 \Lambda_N & \Rightarrow \quad \Lambda_D = 1121 \text{ MeV} \quad \Lambda_B = 1070 \text{ MeV}
  \end{cases}
\end{align*}

S.Yasui and K.Sudoh PRD\textbf{80}(2009)034008
P(*)N Interaction (P(*) = D(*), B(*)): OPEP

\[ V_{\pi PN-P^*N} = -\frac{g_\pi g_{\pi NN}}{\sqrt{2}m_N f_\pi} \frac{1}{3} \left[ \vec{\tau}_P \cdot \vec{\sigma} C(r) + S_\varepsilon T(r) \right] \vec{\tau}_P \cdot \vec{\tau}_N \]

\[ V_{\pi P^*N-P^*N} = \frac{g_\pi g_{\pi NN}}{\sqrt{2}m_N f_\pi} \frac{1}{3} \left[ \vec{T} \cdot \vec{\sigma} C(r) + S_T T(r) \right] \vec{\tau}_P \cdot \vec{\tau}_N \]

S. Yasui and K. Sudoh PRD 80 (2009) 034008

\( C(r) \): Central force, \( T(r) \): Tensor force

▷ \( T(r) \) generates a strong attraction! ⇔ Deuteron
P(*)N Interaction (P(*) = D(*), B(*)) : OPEP

\[ V_{PN-P*N}^{\pi} = -\frac{g_\pi g_{\pi NN}}{\sqrt{2}m_N f_\pi} \frac{1}{3} \left[ \vec{\varepsilon}^{\dagger} \cdot \vec{\sigma} C(r) + S_\varepsilon T(r) \right] \vec{r}_P \cdot \vec{r}_N \]

\[ V_{P*N-P* N}^{\pi} = \frac{g_\pi g_{\pi NN}}{\sqrt{2}m_N f_\pi} \frac{1}{3} \left[ \vec{T} \cdot \vec{\sigma} C(r) + S_T T(r) \right] \vec{r}_P \cdot \vec{r}_N \]

\[ S.Yasui \text{ and K.Sudoh PRD80(2009)034008} \]

\[ C(r) : \text{Central force}, \quad T(r) : \text{Tensor force} \]

\[ \triangledown T(r) \text{ generates a strong attraction! } \leftrightarrow \text{ Deuteron} \]

Deuteron

\[ \begin{array}{c}
\begin{array}{cc}
\pi & \pi \\
N & N \\
\end{array} \\
\begin{array}{cc}
\vec{s} \cdot \vec{q} & \vec{s} \cdot \vec{q} \\
N & N \\
\end{array} \\
\begin{array}{cc}
\vec{s} \cdot \vec{q} & \vec{s} \cdot \vec{q} \\
N & N \\
\end{array} \\
\end{array} \]

\[ ^3S_1 \quad ^3D_1 \]

PN(\( ^2S_{1/2} \)) \text{ - } P* N(\( ^4D_{1/2} \))

Tensor force \( \Rightarrow \) \( ^3S_1 - ^3D_1 \)
Main Subject

- Hadronic molecules formed by Heavy meson-Nucleon with the $\pi$ exchange potential.

$P = \bar{D}(\bar{c}q), B(\bar{b}q) \rightarrow$ No $q\bar{q}$ annihilation!

Genuinely exotic states!

$\iff$ KN molecules have not been found.

($KN$ interaction is repulsion.)
Results of $P^(*)N$ states (2-body)

Bound state and Resonance
- We solve the coupled-channel Schrödinger equations for $PN$ and $P^*N$ channels.
- Interaction: $\pi$, $\rho$, $\omega$ exchange potentials
Numerical results: $\bar{D}N$ and $BN$ for $I = 0$ (2-body)

$\bar{D}N$ and $BN$ states

$J^P = 1/2^\pm, 3/2^\pm, 5/2^\pm$ with $I = 0$

Unit: MeV

Numerical results: $\bar{D}N$ and $BN$ for $I = 0$ (2-body)

$\bar{D}N$ and $BN$ states

- $J^P = 1/2^\pm, 3/2^\pm, 5/2^\pm$ with $I = 0$
- One bound state

---

$\bar{D}N$ states

$\bar{D}^*N$ Threshold

$\bar{D}N$ Threshold

$P = -$ $P = +$

---

Unit: MeV

Numerical results: \( \bar{D}N \) and \( BN \) for \( I = 0 \) (2-body)\n
\( \bar{D}N \) and \( BN \) states

- \( J^P = 1/2^\pm, 3/2^\pm, 5/2^\pm \) with \( I = 0 \)
- One bound state, and resonances in charm

\[ 113.2 - i8.9 \quad 176.0 - i87.4 \]
\[ 148.2 - i5.1 \quad 26.8 - i65.7 \]

\( \bar{D}N \) Threshold

\( \bar{D}^*N \) Threshold

\( E \)

\( P = - \quad P = + \)

- : Bound state
- : Resonance \((E_{re} - i\Gamma/2)\) Unit: MeV

Numerical results: $\bar{D}N$ and $BN$ for $I = 0$ (2-body)

$\bar{D}N$ and $BN$ states

- $J^P = 1/2^\pm, 3/2^\pm, 5/2^\pm$ with $I = 0$
- One bound state, and resonances in charm and bottom sectors!

Many states near the thresholds.

Numerical results: Bound state in $I(J^P) = 0(1/2^-)$

$\bar{D}N$ and $BN$ states

- Expectation values of OPEP in $\bar{D}N$

<table>
<thead>
<tr>
<th>$\bar{D}N$</th>
<th>$\langle V_{\bar{D}N-\bar{D}^*N} \rangle$</th>
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<td>Central</td>
<td>$-2.5$</td>
<td>$1.6 \times 10^{-1}$</td>
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Table: Expectation values of $V_\pi$ ([MeV])

- The tensor force of $\pi$ exchange potential generates a strong attraction. Especially, $\bar{D}N - \bar{D}^*N$ mixing is important.
Numerical results: Bound state in $I(J^P) = 0(1/2^-)$

$\bar{D}N$ and $BN$ states

- Expectation values of OPEP in $\bar{D}N$

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$\bar{D}N$ and $BN$ states

- Expectation values of OPEP in $\bar{D}N$

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The tensor force of $\pi$ exchange potential generates a strong attraction. Especially, $\bar{D}N - \bar{D}^*N$ mixing is important.

- Mixing effects are enhanced in $BN$ due to small $\Delta m_{BB^*}$.

Table: Expectation values of $V_\pi$ ([MeV])

<table>
<thead>
<tr>
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<th>$\langle V_{\bar{B}N-B^*N} \rangle$</th>
<th>$\langle V_{B^*N-B^*N} \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central</td>
<td>$-8.2$</td>
<td>$1.3$</td>
</tr>
<tr>
<td>Tensor</td>
<td>$-90.2$</td>
<td>$-8.3$</td>
</tr>
</tbody>
</table>
Results of $P^{(*)}NN$ states (3-body)

Exotic dibaryon states: $\bar{D}^{(*)}NN$, $B^{(*)}NN$

with $J^P = 0^-, 1^-$ and $I = 1/2$

Bound state and Resonance

- $P^{(*)}N$ interaction: $\pi \rho \omega$ exchanges
- $NN$ interaction: AV8' potential (B. S. Pudliner, et al., PRC56(1997)1720)
Results of $P^{(*)}NN$ states (3-body)

Exotic dibaryon states: $\bar{D}^{(*)}NN$, $B^{(*)}NN$

(1) $\bar{D}^{(*)}$

(2) $\bar{D}^{(*)}$

with $J^P = 0^-, 1^-$ and $I = 1/2$

Wave functions: Gaussian expansion methods

Bound state and Resonance

- $P^{(*)}N$ interaction: $\pi\rho\omega$ exchanges
- $NN$ interaction: AV8' potential (B. S. Pudliner, et al., PRC56(1997)1720)
**Numerical Results:** $\bar{D}^{(*)}NN$ and $B^{(*)}NN$ for $l = 1/2$ (3-body)

$\bar{D}NN$ and $BNN$

- **Bound states** for $J^P = 0^-$ and **Resonances** for $J^P = 1^-$ are found!
  - YY, S. Yasui, and A. Hosaka, NPA 927 (2014) 110

![Graph showing energy levels for Charm and Bottom](image)

- **Charm**
  - $\bar{D}^*NN$ at $111.2 - i9.3$ MeV
  - $\bar{D}NN$ at $-5.2$ MeV

- **Bottom**
  - $B^{*}NN$ at $6.8 - i0.2$ MeV
  - $BNN$ at $-26.2$ MeV
Energy expectation values of the bound states
\( \bar{D}NN \) and BNN

- Energy expectation values

<table>
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<tr>
<th>( \bar{D}(*)NN )</th>
<th>( \langle V_{\bar{D}N-\bar{D}^*N} \rangle )</th>
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<tr>
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<td>-45.6</td>
<td>-1.0</td>
<td>-0.3</td>
</tr>
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<td>—</td>
<td>—</td>
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YY, S. Yasui, and A. Hosaka, NPA 927 (2014) 110
Energy expectation values of the bound states $\bar{D}NN$ and BNN

- Energy expectation values

The bound state of $\bar{D}NN(0^-)$

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YY, S. Yasui, and A. Hosaka, NPA 927 (2014) 110

- Tensor force of $V_{\bar{D}N-\bar{D}^*N}$ generates the strongest attraction.
  $\Rightarrow$ the $\bar{D}^{(*)}N$ force dominates in the bound state, while the $NN$ force plays a minor role.
If $PNN$ channels are switched off...

**Charm**

\[ \bar{D}^* NN \]

$140 \text{ MeV}$

\[ \bar{D} NN \]

$0$

$-5.2$

$0^-$

$1^-$

\[ 111.2 - i9.3 \]

Unit: MeV

**Bottom**

\[ B^* NN \]

$0$

$-26.2$

$0^-$

$1^-$

\[ 6.8 - i0.2 \]

\[ 46 \text{ MeV} \]
If $PNN$ channels are switched off...

- The bound states for $J^P = 0^-$ vanish, because of absence of $PN - P^*N$ mixings.
- For $J^P = 1^-$ channel, the bound states survive.

⇒ **Feshbash resonance!**
Results of $P_Q N$ states ($m_Q \rightarrow \infty$)

$P_Q^{(*)} N \ (m_{P_Q^*} - m_{P_Q} = 0)$

Heavy quark mass limit
In the heavy quark limit, the suppression of the spin-spin force induces the mass degeneracy. In ordinary heavy hadrons, the mass degeneracy is not realized, but in a HQS doublet, it is:

\[
\begin{align*}
J &= j - 1/2 \\
J' &= j + 1/2
\end{align*}
\]

\[\rightarrow \text{HQS doublet}\]

\[(j \neq 0, s_Q = 1/2)\]

\[(j: \text{Total } J \text{ of light degrees of freedom (Brown muck)})\]
In the heavy quark limit, the suppression of the spin-spin force induces the mass degeneracy. In ordinary heavy hadrons,

\[ P_{\text{meson}} \quad P^*_{\text{meson}} \]

\[ m_P \sim m_{P^*} \]

\[ \begin{align*}
  J &= j - 1/2 \\
  J' &= j + 1/2 \\
\end{align*} \quad \rightarrow \text{HQS doublet} \]

\( j \neq 0, \ s_Q = 1/2 \)

(\( j \): Total \( J \) of light degrees of freedom (Brown muck) )

Hadronic molecule (Multi-hadron system)

\[ P_Q \quad q \quad q \quad q \quad N \]

\( s_Q = 1/2 \)

Is the mass degeneracy realized?
To see the mass degeneracy, **New basis** (\(\bar{Q} - [qN]\)) are introduced.


\[ P_Q N \text{ (Hadron) basis} \]

\[
\begin{align*}
P_Q N (\text{Hadron}) \ basis & \\
\end{align*}
\]
To see the mass degeneracy, **New basis** \((\bar{Q} - [qN])\) are introduced.


\(P_Q N\) (Hadron) basis \(\rightarrow\) **Brown muck basis** \((\bar{Q} - [qN]_j)\)

\[
P_Q N (\text{Hadron}) \text{ basis} \quad \Rightarrow \quad \text{Brown muck basis} (\bar{Q} - [qN]_j) \]

\[
\begin{align*}
&\bar{Q} \quad q \\
&P_Q \\
&N
\end{align*}
\]

\[
\begin{align*}
&\bar{Q} \quad q \\
&s_Q = 1/2 \\
&j = J - 1/2, J + 1/2
\end{align*}
\]
To see the mass degeneracy, New basis \((\bar{Q} - [qN])\) are introduced.


\[ P_Q N \ (\text{Hadron) basis} \rightarrow \text{Brown muck basis} \]

\[
\begin{pmatrix}
|P_Q N(2S_{1/2})\rangle_{1/2} \\
|P_Q^* N(2S_{1/2})\rangle_{1/2} \\
|P_Q^* N(4D_{1/2})\rangle_{1/2}
\end{pmatrix}
= U_{1/2^-} \cdot \begin{pmatrix}
|[qN(1S_0)]\bar{Q}\rangle_{1/2} \\
|[qN(3S_1)]\bar{Q}\rangle_{1/2} \\
|[qN(3D_1)]\bar{Q}\rangle_{1/2}
\end{pmatrix}
\]
To see the mass degeneracy, New basis ($\bar{Q} - [qN]$) are introduced.


$P_Q N$ (Hadron) basis

$\rightarrow$

Brown muck basis ($\bar{Q}-[qN]_j$)

Brown muck

\[ \begin{align*}
    j &= J - 1/2, J + 1/2 \\
    &= 0, 1 \text{ (for } J^P = 1/2^-) 
\end{align*} \]

Basis are transformed by a unitary matrix $U$: for $J^P = 1/2^-$

\[
\begin{pmatrix}
    |P_Q N(2S_1/2)\rangle_{1/2} \\
    |P_Q^* N(2S_1/2)\rangle_{1/2} \\
    |P_Q^* N(4D_1/2)\rangle_{1/2}
\end{pmatrix} = U_{1/2^-} \begin{pmatrix}
    |[qN(1S_0)]\bar{Q}\rangle_{1/2} \\
    |[qN(3S_1)]\bar{Q}\rangle_{1/2} \\
    |[qN(3D_1)]\bar{Q}\rangle_{1/2}
\end{pmatrix}
\]
To see the mass degeneracy, New basis (\(\bar{Q} - [qN]\)) are introduced.


\[ P_QN \text{ (Hadron) basis} \]

\[ \Rightarrow \]

\[ \text{Brown muck basis (}\bar{Q}[-qN]_j) \]

\[ j = J - 1/2, J + 1/2 \]

\[ = 0, 1 \text{ (for } J^P = 1/2^-) \]

\[ s_Q = 1/2 \]

\[ \begin{pmatrix}
    |P_QN(2S_{1/2})\rangle_{1/2} \\
    |P^*_QN(2S_{1/2})\rangle_{1/2} \\
    |P^*_QN(4D_{1/2})\rangle_{1/2}
\end{pmatrix} =
\begin{pmatrix}
    -1/2 & \sqrt{3}/2 & 0 \\
    \sqrt{3}/2 & 1/2 & 0 \\
    0 & 0 & -1
\end{pmatrix}
\begin{pmatrix}
    |[qN(1S_0)]\bar{Q}\rangle_{1/2} \\
    |[qN(3S_1)]\bar{Q}\rangle_{1/2} \\
    |[qN(3D_1)]\bar{Q}\rangle_{1/2}
\end{pmatrix} \]
By introducing the brown muck basis, Hamiltonian $H(= K + V_\pi)$ is transformed.

$$H_{1/2^-} = \begin{pmatrix}
    K_0 & \sqrt{3}C & -\sqrt{6}T \\
    \sqrt{3}C & K_0 - 2C & -\sqrt{2}T \\
    -\sqrt{6}T & -\sqrt{2}T & K_2 + C - 2T
\end{pmatrix}$$

* $K_i$: Kinetic term, $C$: Central force, $T$: Tensor force

In the Brawn muck basis,
By introducing the brown muck basis, Hamiltonian $H(= K + V_{\pi})$ is transformed.

$$H_{1/2^-}^{BM} = \begin{pmatrix}
K_0 - 3C & 0 & 0 \\
0 & K_0 + C & 2\sqrt{2}T \\
0 & 2\sqrt{2}T & K_2 + C - 2T
\end{pmatrix}$$

* $K_i$: Kinetic term, $C$: Central force, $T$: Tensor force

In the Brawn muck basis, $H_{1/2^-}$ is **block diagonalized**!

Components of $j = 0$ and 1 ($j$: total $J$ of Brown muck)
By introducing the brown muck basis, Hamiltonian \( H(= K + V_\pi) \) is transformed.

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In the Brawn muck basis, \( H_{1/2^-} \) is **block diagonalized**!

Components of \( j = 0 \) and 1 (\( j \): total \( J \) of Brown muck)

For \( J^P = 3/2^- \),

\[
H_{3/2^-} = \begin{pmatrix}
K_2 & \sqrt{3}T & -\sqrt{3}T & \sqrt{3}C \\
\sqrt{3}T & K_0 + C & 2T & T \\
-\sqrt{3}T & 2T & K_2 + C & -T \\
\sqrt{3}C & T & -T & K_2 - 2C
\end{pmatrix}
\]
Hamiltonian \((m_Q \rightarrow \infty)\) in the Brawn muck basis

- By introducing the brown muck basis, Hamiltonian \(H(= K + V_\pi)\) is transformed.

\[
H_{1/2^-}^{BM} = \begin{pmatrix}
K_0 - 3C & 0 & 0 \\
0 & K_0 + C & 2\sqrt{2}T \\
0 & 2\sqrt{2}T & K_2 + C - 2T
\end{pmatrix}
\]

\[
H_{3/2^-}^{BM} = \begin{pmatrix}
K_0 + C & 2\sqrt{2}T & 0 & 0 \\
2\sqrt{2}T & K_2 + C - 2T & 0 & 0 \\
0 & 0 & K_2 - 3C & 0 \\
0 & 0 & 0 & K_2 + C + 2T
\end{pmatrix}
\]

* \(K_i\): Kinetic term, \(C\): Central force, \(T\): Tensor force

- In the Brawn muck basis, \(H_{1/2^-}\) is **block diagonalized**! Components of \(j = 0\) and \(1\) (\(j\): total \(J\) of Brown muck)

- For \(J^P = 3/2^-\), \((j = 1, 2)\)
Hamiltonian \((m_Q \rightarrow \infty)\) in the Brawn muck basis

- By introducing the brown muck basis, Hamiltonian \(H(= K + V_\pi)\) is transformed.

\[
H_{1/2^-}^{BM} = \begin{pmatrix}
K_0 - 3C & 0 & 0 \\
0 & K_0 + C & 2\sqrt{2}T \\
0 & 2\sqrt{2}T & K_2 + C - 2T
\end{pmatrix}
\]

* \(K_i\): Kinetic term, \(C\): Central force, \(T\): Tensor force

- In the Brawn muck basis, \(H_{1/2^-}\) is **block diagonalized**!
- Components of \(j = 0\) and \(1\) (\(j\): total \(J\) of Brown muck)

- For \(J^P = 3/2^-\), \((j = 1, 2)\)

\[
H_{3/2^-}^{BM} = \begin{pmatrix}
K_0 + C & 2\sqrt{2}T & 0 & 0 \\
2\sqrt{2}T & K_2 + C - 2T & 0 & 0 \\
0 & 0 & K_2 - 3C & 0 \\
0 & 0 & 0 & K_2 + C + 2T
\end{pmatrix}
\]

- Components of \(j = 1\) are same! Doublet for \(1/2^-\), \(3/2^-\)
Numerical Results: PN molecule in $m_Q \rightarrow \infty$

- In the $\bar{D}N$ and $BN$ sectors (with finite heavy quark mass), bound states ($J^P = 1/2^-$) and resonances ($3/2^-$) were found.

![Diagram showing energy levels and assignments for $\bar{D}N$, $BN$, and $P_{QN}$ with particles $\bar{c}$, $\bar{b}$, and $\bar{Q}$ labeled.]
In the $\bar{D}N$ and $BN$ sectors (with finite heavy quark mass), Bound states ($J^P = 1/2^-$) and resonances ($3/2^-$) were found.

Degenerate states are found! ($1/2^-$ and $3/2^-$)

In the $\bar{D}N$ and $BN$ sectors (with finite heavy quark mass), Bound states ($J^P = 1/2^-$) and resonances ($3/2^-$) were found.

Degenerate states are found! ($1/2^-$ and $3/2^-$) ⇒ They belong to the HQS doublet as expected.

Numerical Results: $P^{(*)}NN$ states ($m_Q \rightarrow \infty$)

- Energy-levels for $\bar{D}NN$, $BNN$ and $P_QNN$ ($m_Q \rightarrow \infty$)

- $PNN$ states with $J^P = 0^-$ and $1^-$ are degenerate!

- In the experiments, doublet states of heavy meson nuclei are expected!
Summary

- We have investigated hadronic molecules $P(\ast)N$ and $P(\ast)NN$ with respecting the Heavy Quark Spin Symmetry.
- Many bound states and resonances are found in $P(\ast)N$ and $P(\ast)NN$.
- **Tensor force of OPEP in PN — P*N mixing** plays a crucial role to produce a strong attraction.
- In $m_Q \to \infty$, we have obtained the degenerate states in the hadronic molecule by introducing the Brown basis. The degeneracy has found in numerical results.

**Thank you for your kind attention.**
Back up
Heavy-light chiral lagrangian

\[ \mathcal{L}_{\pi HH} = -\frac{g_\pi}{\sqrt{2}f_\pi} \text{Tr} \left[ H_b \gamma_\mu \gamma_5 \vec{\tau} \cdot \partial^\mu \pi_{ba} \bar{H}_a \right] \]

\[ \mathcal{L}_{\nu HH} = -i\beta \text{Tr} \left[ H_b \nu^\mu (\rho_\mu)_{ba} \bar{H}_a \right] + i\lambda \text{Tr} \left[ H_b \sigma^{\mu\nu} F_{\mu\nu}(\rho)_{ba} \bar{H}_a \right] \]

Heavy meson field

\[ H_a = \frac{1+\gamma^5}{2} \left[ P_a^{*\mu} \gamma^\mu - P_a \gamma^5 \right], \quad \bar{H}_a = \gamma^0 H_{a\dagger} \gamma^0 \]

vector pseudoscalar

Bonn model

\[ \mathcal{L}_{\pi NN} = ig_{\pi NN} \bar{N}_b \gamma^5 N_a \vec{\tau} \cdot \vec{\pi}_{ba} \]

\[ \mathcal{L}_{\nu NN} = g_{\nu NN} \bar{N}_b \left( \gamma^\mu (\hat{\rho}_\mu)_{ba} + \frac{\kappa}{2m_N} \sigma_{\mu\nu} \partial^\nu (\hat{\rho}^\mu)_{ba} \right) N_a \]

These coupling constants are fixed! 11/4, 2014 Y. Yamaguchi (RIKEN)
Lagrangian($P(\ast) - N$); $\pi$, $\rho$ and $\omega$ exchanges

### Heavy-light chiral lagrangian

- $L_{\pi HH} = - \frac{g_\pi}{\sqrt{2} f_\pi} \text{Tr} \left[ H_b \gamma_\mu \gamma_5 \vec{\tau} \cdot \partial^\mu \vec{\pi}_{ba} H_a \right]$ 
  From $D^* \rightarrow D\pi$ decay

- $L_{\nu HH} = - i \beta \text{Tr} \left[ H_b \nu^\mu (\bar{\rho}_\mu)_{ba} H_a \right] + i \lambda \text{Tr} \left[ H_b \sigma^{\mu\nu} F_{\mu\nu}(\bar{\rho})_{ba} H_a \right]$ 
  From radiative and semileptonic decays of heavy meson

#### Heavy meson field

$H_a = \frac{1 + \gamma^\gamma}{2} \left[ P^*_a \gamma^\gamma - P_a \gamma^5 \right]$, $\bar{H}_a = \gamma^0 H^+_a \gamma^0$

- vector
- pseudoscalar

### Bonn model

- From NN data

  - $L_{\pi NN} = i g_{\pi NN} \tilde{N}_b \gamma^5 N_a \vec{\tau} \cdot \vec{\pi}_{ba}$

  - $L_{\nu NN} = g_{\nu NN} \tilde{N}_b \left( \gamma^\mu (\tilde{\rho}_\mu)_{ba} + \frac{\kappa}{2m_N} \sigma_{\mu\nu} \partial^\nu (\tilde{\rho}^\mu)_{ba} \right) N_a$

**These coupling constants are fixed!**
Central force and Tensor force

- Central force $C(r)$ and Tensor force $T(r)$

\[
C(r) = \int \frac{d^3 q}{(2\pi)^3} \frac{m^2_\pi}{q^2 + m^2_\pi} e^{i\vec{q} \cdot \vec{r}} F(\Lambda_P, \vec{q}) F(\Lambda_N, \vec{q})
\]

\[
S_T(\vec{r}) T(r) = \int \frac{d^3 q}{(2\pi)^3} \frac{-\vec{q}^2}{q^2 + m^2_\pi} S_T(\vec{q}) e^{i\vec{q} \cdot \vec{r}} F(\Lambda_P, \vec{q}) F(\Lambda_N, \vec{q})
\]

\[
F(\Lambda, \vec{q}) = \frac{\Lambda^2 - m^2_\pi}{\Lambda^2 + \vec{q}^2}
\]

- Spin operators

\[
\vec{\epsilon}^{\pm} = (\mp \frac{1}{\sqrt{2}}, \pm i \sqrt{2}, 0), \quad \vec{\epsilon}^{(0)} = (0, 0, 1)
\]

\[
T^i = i \epsilon^{ijk} \epsilon^j \epsilon^k
\]

- Coupling constants

\[
g_\pi = 0.59 \text{ for } \bar{D} \text{ and } B, \quad g_{\pi NN}^2 / 4\pi = 13.6
\]

\[
(g_{D^*D} = 2\sqrt{m_D m_{D^*}} g_\pi / f_\pi = 17.9)
\]
\[ J^P = 1/2^- \ (j = 0, 1) \]
\[
\begin{pmatrix}
\mathbf{P_QN}^{(2S_1/2)}_{1/2} \\
\mathbf{P_QN}^{(2S_1/2)}_{1/2} \\
\mathbf{P_QN}^{(4D_1/2)}_{1/2}
\end{pmatrix}
= \begin{pmatrix}
-1/2 & \sqrt{3}/2 & 0 \\
\sqrt{3}/2 & 1/2 & 0 \\
0 & 0 & -1
\end{pmatrix}
= U_{1/2^-}
\]

\[ J^P = 3/2^- \ (j = 1, 2) \]
\[
\begin{pmatrix}
\mathbf{PN}^{(2D_3/2)} \\
\mathbf{P_N}^{(4S_3/2)} \\
\mathbf{P_N}^{(4D_3/2)} \\
\mathbf{P_N}^{(2D_3/2)}
\end{pmatrix}
= \begin{pmatrix}
0 & \sqrt{6}/4 & 1/2 & \sqrt{6}/4 \\
1 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\
0 & \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2} & \frac{1}{2\sqrt{2}}
\end{pmatrix}
\]

ハミルトニアンの変換
\[ H_{JP} \rightarrow H^{BM} = U_{JP}^{-1} H_{JP} U_{JP} \]